UDC 531.55 :521.1

QUALITATIVE STUDY OF PROPERTIES OF MOTION OF A SATELLITE OF A SPHEROIDAL PLANET

PMM Vol. 41, № 3, 1977, pp. 561-564

M. Kh. KHASANOVA
(Dushanbe)

(Received May 20, 1976)

Qualitative investigation of motion of artificial satellites in a noncentral axisymmetric gravity field is carried out to within the fourth zonal harmonic.

Taking into account the fact that the geocentric longitude of the satellite is a cyclic coordinate and utilizing the Routh method of ignoring the cyclic coordinates, we can write the equations of motion of the satellite in the form

$$\rho^{\cdot\cdot} = dW / d\rho, \quad z^{\cdot\cdot} = dW / dz, \quad W = U - {}^1/_2\, C^2 / \rho^2$$

Here ρ and z are cylindrical coordinates, U is the Earth's gravitational potential[1] and C is the constant of the area integral, corresponding to the cyclic coordinate.

First we shall investigate the domains in which motions of the satellite are feasible. These domains are bounded by the Hill surface

$$W(\rho, z) + h = 0 \tag{1}$$

which follows obviously from the integral of kinetic energy $v^2 = 2(W + h)$, i.e. from the condition that the square of the velocity of the satellite is nonnegative. Substituting in (1), in place of the force function, the expression [1]

$$\frac{fM}{r}\left[1 - \sum_{k=2}^{\infty} I_k \left(\frac{R}{r}\right)^k P_k(\sin\varphi)\right]$$

in which f is the gravitational constant, M is the Earth's mass, R is its mean equatorial radius, r is the geocentric distance of the satellite, I_k are dimensionless constants depending on the distribution of the Earth's masses, $P_k(\sin\varphi)$ are the Legendre polynomials and φ is the geocentric latitude, we obtain, for $I_k = 0$, the equation of the Hill surface in the following form:

$$r_0 = a\,(1 \mp \sqrt{1 - \cos^2 i \sec^2 \varphi}), \quad \cos i = \frac{c\sqrt{-2h}}{fM}$$

$$\left(h = -\frac{fM}{2a} < 0\right)$$

where i is the angle of inclination of the satellite orbit to the Earth's equatorial plane. If $I_k \neq 0$, we assign to I_k the necessary values to obtain the domains of possible motions which can be used to assess the influence of the Earth's flattening on the motion of the satellite. Retaining in the expression for the Earth's gravitational potential the second zonal harmonic only, we obtain the equation of the Hill surface in the form

$$r = r_0 + r_1 I_2 + r_2 I_2^2 + \cdots$$

Setting $I_3 = \alpha I_2^2$, $I_4 = \beta I_2^2$ we shall seek the solutions of the above equation in the form of a series of powers of the small parameter I_2. This yields the equation of the Hill surface in the form

$$r_0 - r_1 I_2 - r_2 I_2^2 + \left[\frac{R^3 (5\sin^2\varphi - 3)\sin 2\varphi - \cos\varphi}{4r_0 aS(i,\varphi)}\right] I_3 +$$
$$\left\{\frac{R^4 [5\sin^2\varphi (7\sin^2\varphi - 6) + 3]\cos^2\varphi}{8a^3 S(i,\varphi)}\right\} I_4 = 0$$
$$r_1 = \frac{R^2 (3\sin^2\varphi - 1)\cos^2\varphi}{2aS(i,\varphi)}$$
$$r_2 = \frac{r_1^2}{2r_0}\left[6 + \frac{2b\cos^2\varphi - 3(1-e^2)\cos^2 i}{S(i,\varphi)}\right]$$
$$C^2 = f\mu a(1-e^2)\cos^2 i, \quad b = r_0/a, \quad S(i,\varphi) = [(1-e^2)\cos^2 i - b\cos^2\varphi]$$

Let us investigate the influence of the second zonal harmonic and the combined effect of the first three zonal harmonics. Consider the following quantities

$$\Delta r' = r_1 I_2, \quad \Delta r'' = r_1 I_2 + r_2 I_2^2 + \cdots$$

To estimate the effect of the Earth's oblateness on the domains of possible motions of the satellite, we use the following zonal harmonics: I_2, I_3, I_4 [2]. The computations were carried out on a Minsk-22 computer for the flight altitudes ranging from 200 to 1000 km, in 100 km steps. The domain of possible motions was investigated

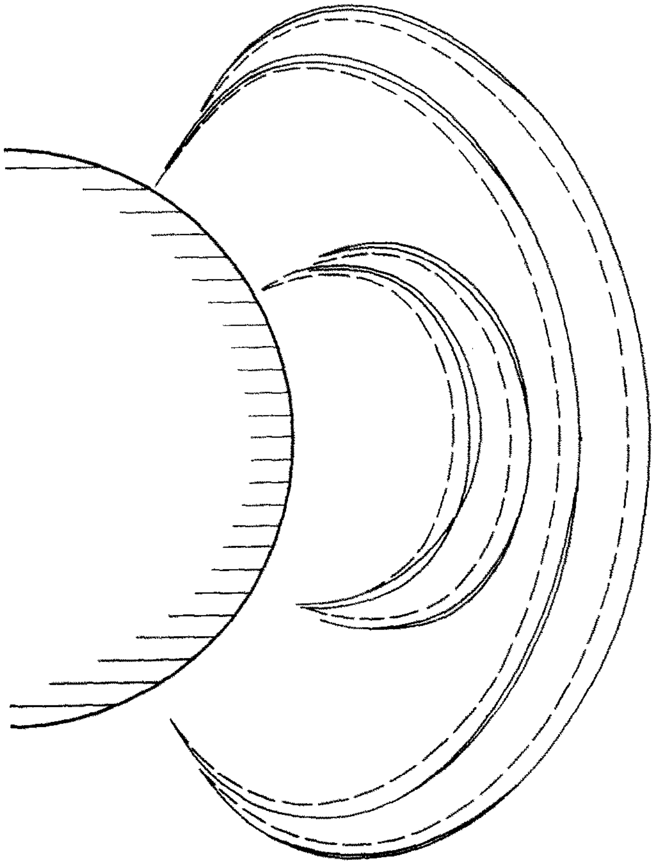

Fig. 1

for the angles of inclination of the satellite orbit to the equatorial plane ranging from 0° to 180° in 30° steps, and for various values e of the osculating eccentricity of the satellite ($e = 0, 0.2$). The perturbing character of the zonal harmonics of various orders can be assessed from the form of the domain of possible motions.

Fig. 1 depicts the domains of possible motions of a satellite in the polar r, φ-coordinate system, for $e = 0.2$, $i = 30°, 60°$ and for the flight altitudes of 200 km and 1000km. The dashed line depicts the Hill curve for the case of a Keplerian motion, and the solid line corresponds to a perturbed motion. Fig. 2 shows the graphs of the increments in the radius vector of the limiting Hill surface versus the oblateness of the Earth. The dashed line in Fig. 2 defines $\Delta r'$ and the solid line $\Delta r''$.

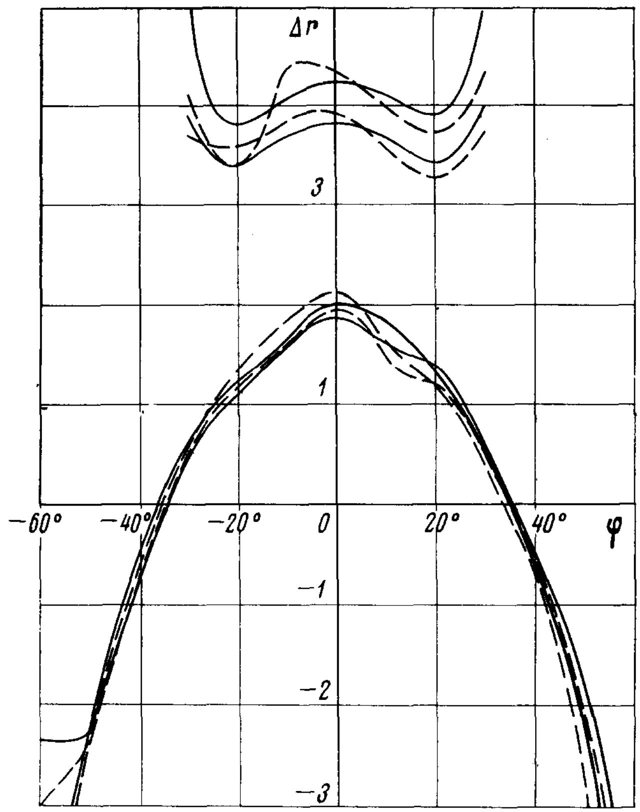

Fig. 2

REFERENCES

1. Abalakin V. K., Aksenov E. P., Grebenikov E. A. and Riabov Iu. A., Celestial Mechanics and Astrodynamics Reference Book. Moscow, "Nauka", 1971.

2. Kozai Y., The potential of the earth derived from satellite motions. In: Dynamics satellites. Sympos, Paris, 1962, Berlin, Springer-Verlag, 1963.

Translated by L. K.